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A New Theory of Elementary Matter.

Part I: Philosophical Approach and General Implications

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Abstract

This is the first of a series of articles that reviews and expands upon a new theory of elementary matter. This paper presents an exposition of the philosophical approach and its general implications. The ensuing explicit form of the mathematical expression of the theory and several applications in the atomic and elementary particle domains will be developed in the succeeding parts of this series.

The theory is based on three axioms: the principle of general relativity, a generalized Mach principle, and a correspondence principle. The approach is basically a deterministic, relativistic field theory which fully incorporates the idea that any realistic physical system is in fact *a closed system*, without separable parts. It is shown that the most primitive mathematical expression of this theory, following as a *necessary consequence* of its axioms, is in terms of a set of coupled nonlinear spinor field equations. Nevertheless, the exact formalism is constructed to asymptotically approach the quantum mechanical formalism for a many-particle system, in the limit of sufficiently small energy-momentum transfer among the components of the considered closed system. Thus, all of the mathematical predictions of nonrelativistic quantum mechanics are contained in this theory, as a mathematical approximation. However, predictions follow from the exact form of this theory (where energy-momentum transfer can be arbitrarily large) that are not contained in the quantum theory.

1. *Introduction*

Among the different periods in the history of physics when discontinuous conceptual innovations appeared, the twentieth century has been unique in its simultaneous introduction of *two* revolutions in science—the theory of relativity and the quantum theory of measurement. During the first quarter of this century, when these innovations first appeared, it was not necessary, in explaining most of the experimental results, to describe the two theories in a unified form. Indeed, it was adequate to use the quantum theory to describe the nonrelativistic behavior of microscopic matter (e.g., atoms, molecules, electrons) and to use the theory of special relativity to describe electromagnetic radiation or macroscopic (or microscopic) quantities of matter that move at speeds comparable with the speed of light, so long as their quantum behavior could be ignored.

Nevertheless, it soon became necessary to fuse these two theories into a single general theory. This was to adequately describe two different types of physical phenomena. The first had to do with the full treatment of radiating matter, such as the decay of excited atoms or nuclei. While the matter constituents of the gas of excited atoms that produce the observed spectra may, under the proper circumstances, be treated as nonrelativistic entities, they are indeed emitting electromagnetic radiation—an entity that has no nonrelativistic limit! Thus, the full description of the emission and absorption of radiation by atoms necessarily requires a relativistic treatment.

The second class of data that necessitated the fusion of the quantum and relativity theories was the result of experimental arrangements that enable the detection of the effects of very high-speed material particles, either from natural sources (e.g. cosmic rays) or from the output of man-made nuclear accelerators.

Recognizing this theoretical necessity, physics introduced *quantum field theory*—a formalism that attempts to extend the notions of the quantum theory so as to be compatible with the *mathematical* requirements of the theory of special relativity. While there have been some remarkable numerical successes in the predictions of this theory, it is still not in as satisfactory a state as required, primarily because of the fact that in its present exposition, it has not been possible to demonstrate the existence of well-defined mathematical solutions.

Many feel that this is strictly a mathematical difficulty and does not really challenge the conceptual foundation of present-day thinking about the fundamental nature of matter. This conclusion may very well be true. Still, so long as the problem has not yet been resolved, it must be admitted by the objective scientist that the difficulty could, in fact, be rooted in the aspects of the conceptual foundations of the formalism that have to do with the quantum theory of measurement.

Taking the latter view, this author has been investigating an alternative approach to the properties of matter. This is a theory that would accommodate high-energy and low-energy physics within a common theoretical structure that does not entail any special demarcation between the macroscopic and microscopic domains.

The quantum field theory of the present-day approach fully adopts the conceptual notions of the quantum theory of measurement (discrete particle concept, nondeterminism, discontinuity, linearity, open system) while attempting to incorporate only *the mathematical requirement of relativity theory*—that all statements about elementary particles and their interactions must be Lorentz invariant. The theory developed by this author takes the opposite view—that of fully adopting the conceptual notions of the theory of relativity (the continuous field concept, determinism, nonlinearity, closed system) while incorporating *the mathematical requirement that the formalism should asymptotically approach that of the quantum theory* in the nonrelativistic domain (i.e., the imposition of this correspondence principle).

In the next section, the axioms of this theory will be stated explicitly and discussed. In Section 3, the philosophical and general mathematical implications of these axioms will be derived. Part II of this series (see p. 453)

deals with the interpretation and the most general formalism of electromagnetic theory, according to the axiomatic structure of this theory. It will be seen that, philosophically, there is an overlap in notions of this field theory and the relativistic action-at-a-distance approach to particle theories, proposed by several previous authors.† It will also be shown in this section how this approach leads to a true generalization of the Maxwell formalism—a factorization from the vector representation of the theory to a first-rank spinor representation—which in turn leads to predictions in electrodynamics that are out of the domain of predictions of the conventional theory.

Also in Part II, the 'matter field equations' of this theory will be derived from the consideration of a special mapping between reflected 2-component spinor variables in a Riemannian space. The general, nonlinear form of these field equations, for a closed system, will be demonstrated, and shown to approach the Dirac form of the quantum mechanical equations, in the appropriate limit, which, in turn, approach the standard Schrödinger form of quantum mechanical equation, in the limit of nonrelativistic physics. An important result in the general analysis, is the derivation of a positive-definite field, appearing in the place in these equations where the inertial mass parameter is normally inserted. Three important features of this *derived* inertial mass field are that (1) it implies that gravitational forces can only be attractive; (2) its averaged values, in the asymptotic limit where there is vanishingly small energy-momentum transfer between the components of a closed system, *approach* a discrete spectrum; and (3) the inertial mass of any quantity of matter tends to zero, as all of the other matter of the closed system should be depleted—in accordance with the Mach principle (one of the axioms of the theory). It will also be proven as a consequence of the imposition of gauge invariance on the matter field equations, that electromagnetic forces can be attractive or repulsive, depending on the features of the geometry of the space-time.

In Part III (Sachs, 1971b), it will be shown how the nonlinear field equations of this theory approach the standard quantum mechanical equations for a many-particle system (the Hartree–Fock formalism). In this section, a proof will be carried out showing that, as an exact feature of the nonlinear, relativistic field equations of this theory of a closed system, physical implications follow that are identical with the assertion of the *Pauli exclusion principle*. It will then be shown with this result that in the nonrelativistic limit, the fundamental field that describes the mutual interaction of the closed system of identical (spinor) matter components approaches the many-body, antisymmetrized wave function of quantum mechanics—the form that conventionally underlies Fermi-Dirac statistics for a many-particle system.

† From the earliest periods of modern physics (in the 1920's) until the present day, many different authors have written on this general approach. A sample of papers in this area are those of Tetrode (1922), Lewis (1926), Wheeler & Feynman (1945) and Davies (1970). Bibliography on other papers in this field may be found in these authors' works.

In Part IV (Sachs, 1971c), the exact form of the field equations for the closed system will be applied to the 2-body particle–antiparticle pair. An *exact solution* for the electron–positron system will be demonstrated, that predicts *all* of the experimental observations that are conventionally interpreted in terms of pair 'annihilation' and 'creation'. However, matter is not destroyed or created in this theory. The solution discovered relates to the ground state (maximum binding) for the pair. In this state, the pair does not readily give up energy and momentum to its surroundings (say, the atoms in a bubble chamber)—thereby giving the *appearance* of annihilation. The results of experimentation which lead to the dynamical and kinematic properties of the system, when annihilation or creation are supposed to take place, are explained here without the need to introduce the 'photon' concept. With this result, it is then argued that any region of space should be populated with some (definite) number of particle–antiparticle pairs, each in their ground states of minimum energy-momentum (maximum binding). With the assumption that a cavity contains an *ideal gas* of such pairs, and the dynamical properties of the pairs that have been derived from the field equations, the Planck spectral distribution for blackbody radiation is then demonstrated—again without the need of the 'photon' concept. It is concluded that the 'photon' concept is superfluous in electrodynamics.

Also in Part IV, the theory is applied to the electron–proton system. It is found that the entire hydrogen spectrum is predicted, in agreement with the data. This includes the Lamb splitting. The result follows from a natural generalization of the Coulomb law, as a consequence of the factorization of the vector representation of electromagnetic theory into a 2-component spinor representation. The hydrogen spectrum, then, is in this case a property of the *closed* electron–proton system. There is no need to bring in a 'physical vacuum', composed of annihilating and creating pairs and radiation, as it is done in quantum electrodynamics, to explain the Lamb shift. Also in contrast with the conventional theory, this theory derives these results from a demonstrably mathematically consistent formalism, which has bona fide solutions and where no quantities become infinite at any stage of the calculations. Finally, it is shown that with an excited hydrogen atom in a background 'ideal gas' of electron–positron pairs, in their ground states (derived in the preceding section), the latter act as an 'absorber' in the de-excitation of the atom, predicting the same lifetimes of the excited states of hydrogen as is observed.

Summing up, this series of papers presents the results of a new approach to the theory of matter in the microscopic domain that fully incorporates the idea of a *closed system* with a relativistic field theory. The approach is deterministic (i.e., all of the features of the system are 'predetermined'—they are independent of any measurements that may or may not be performed), nonlinear and based on a principle of continuity within the field approach of relativity theory. The formalism is constructed to obey a correspondence principle—approaching the equations of quantum mechanics in the limit of small energy-momentum transfer within the system of matter components of a closed physical entity. But with its exact nonlinear structure, the following *derivations* follow, from first principles: (1) the Pauli exclusion principle; (2) the inertial mass of matter; (3) a bound state of the particle-antiparticle pair that predicts all of the experimental evidence which is conventionally interpreted as pair 'annihilation' and 'creation'; (4) the spectral distribution of blackbody radiation; and (5) the correct properties of the hydrogen spectrum, including the Lamb splittings and the lifetimes of its excited states.

2. *Axiomatic Basis*

Axiom 1. The principle of relativity asserts that the laws of nature must be independent of the frame of reference in which they are expressed. The 'frame of reference' here refers to a particular system of space-time coordinates that is distinguishable from other space-time coordinate systems only in terms of their *relative motion.* When the relative motion happens to be characterized by constant, rectilinear speed, the theory reduces to the special case that is called 'special relativity theory'. With the more general type of motion, it is called 'general relativity theory'.

Axiom 2. The generalized Mach principle asserts that there are no intrinsic properties of 'free' matter—that *all* of the manifestations of any (apparently free) quantity of matter are in fact measures of the mutual dynamical coupling within a closed material system.

Mach's original assertion referred to the particular manifestation of matter that is associated with its inertia—the resistance with which matter opposes a change in its state of motion by the application of external forces. To review his argument, first consider the classical interpretation of inertia of Newton and Galileo. According to this view, the inertial mass of matter is one of its intrinsic properties. It follows from the (empirically confirmed) law that if F_1 and F_2 are the magnitudes of two different external forces that act on a given bit of matter and if a_1 and a_2 are the magnitudes of the respective accelerations caused by these forces, then $F_1/F_2 = a_1/a_2$. Another way to state this conclusion is in terms of the usual expression of Newton's second law of motion, $F = ma$, where m is the constant of proportionality between F (the cause) and a (the effect). The constant m, then, was taken to quantify the inertia of matter, as it was defined by Galileo in his assertion of *the principle of inertia.*

In contrast, Mach argued that it is still consistent with the empirically verified law, $F = ma$, to interpret it as pertaining to a linear relation between the ratio of forces that cause different bodies, with masses m_1 and m_2, to accelerate at the same rate a and the ratio of these masses. That is to say, if $a_1 = a_2 = a$ for two different bodies, then $F_1/F_2 = m_1/m_2$. From this, one has the relation $m_1 = kF_1$ where $k = (m_2/F_2)$ may be taken as a *standard.*

From the conceptual point of view, one may interpret this equation to say that the source of the inertial mass m_1 is rooted in the dynamical coupling between this body and all of the other bodies in a closed system—as expressed in terms of the total external force F_1 that acts on m_1. Thus, Mach's conclusion about the origin of the inertial manifestation of matter is opposite to that of Newton and Galileo. The latter view is atomistic—it is based on a property of a free, non-interacting bit of matter. Mach's view of the inertia of matter is anti-atomistic. It is rather a feature of a closed system of matter —a system in which no matter is 'free' of the other matter in its surroundings. This view further implies that the inertial features of matter relate to the entire system, rather than to absolute attributes of the components of a material system.

It is well known that Mach's view of inertia had a profound influence on Einstein's development of the theory of relativity. In this article, it is contended that a full exploitation of the relativistic view implies that not only the inertial manifestations of matter, but all of its manifestations must, in fact, be related to the mutual dynamical coupling of the components of a *closed system*. This assertion is referred to here as the *Generalized Mach Principle* (Sachs, 1969a, 1970a). It will be applied, in particular, to the electrodynamic and gravitational, as well as the inertial manifestations of interacting matter.

One of the important implications of the generalized Mach principle when incorporated with the theory of relativity (i.e. when we combine Axioms 1 and 2) is that the components of an interacting system of matter lose meaning as *things-in-themselves*. It is only the entire closed system that has meaning here as a fundamental entity. Thus, the conceptual approach of this theory starts with a system that has at least two components—*two* because of the empirical fact that (in particular limits) it looks as though there are at least two separate and distinguishable entities which are free of each other. According to the theory discussed here, however, the mutual coupling between these components can be arbitrarily weak, but it may never be 'off'. This is a point with more than philosophical significance. It is a feature of the theory that also has mathematical consequences that differ from those of a theory of free things that later can be considered as interacting weakly. Thus (at least some of) the predictions of the two approaches, in regard to the experimental observations, would necessarily be different.

According to the *Generalized Mach Principle* in relativity theory, then, the 'observer–observed' relation is not to be taken as a coupling between the independent things—'observer' and 'observed'—since here there is no meaning to the term 'observer' as a thing by itself, or the term 'observed', as a thing by itself. That is to say, 'observer' is only meaningful as an entity that is relative to 'observed' and vice versa. It is the *whole entity*, 'observer–observed', which, in this view is inseparable (*in principle*) that is the fundamental unit from which one must start to construct a general theory of matter.

It is interesting to note that, contrary to Mach's original positivistic view, 'observer' here does not necessarily refer to a *human* observer, or to his equipment. Neither does it necessarily refer to a macroscopic entity whenever the 'observed' refers to a microscopic entity (as assumed in the quantum theory). Rather, 'observer–observed' refers to a closed system that is fundamentally one. It is sometimes necessary to identify a component of the closed system with a macroscopic measuring apparatus and the remainder of the system with the 'observed' atom. However, these identifications can only come after the limiting form of a general system (that does not make such distinctions) has been taken. In other examples, one of the interacting components of the closed system may be an electron and the other a positron—i.e., here there is no macroscopic component in the initial system whose properties are being determined. The important requirement of the theory is that *the overall description must be independent of which component of the closed system is called 'observer' and which is called 'observed'*.

Since the 'observer' is the subject and the 'observed' the object of any statement about a material system, and since the overall description must be invariant with respect to an interchange of the object and subject components of the system, it follows that such an approach presents a completely objective description of matter—the fundamental description is independent of the nature of the observations in any particular measurement. That is to say, all of the features of a material system are *predetermined*. This view takes the philosophical stand of realism. It is in contrast with the non-determinism and the philosophical stand of positivism that is taken in the Copenhagen interpretation of the equations of quantum mechanics. It is also in contrast with Mach's original positivistic interpretation of Newton's equations of motion.

Axiom 3. The correspondence principle asserts that the expression of a new theory must *approach* the mathematical formalisms of the theories that it attempts to supersede. Thus, to approach the quantum mechanical equations in the microscopic domain, the equations of the new theory must be taken to be differential equations that would approach the usual Hamiltonian form in the proper limit. The parameters that characterize the asymptotic limit where the correspondence between a fully relativistic theory and the earlier formalism applies are the numbers which measure the quantity of energy-momentum transfer between interacting matter. This is the limit when the system *appears to be* in terms of distinguishable 'parts'.

It will be found that the axioms of this theory lead to a mathematical formalism that is a *deterministic, nonlinear* spinor field theory—thus containing features that are not at all in correspondence with the present-day expression of high energy physics in the elementary particle domain. However, this formalism, to be consistent with Axiom 3, does approach that of ordinary quantum mechanics and electromagnetic theory and the other conventional formalisms, when the energy-momentum transfer between matter and matter becomes sufficiently small. The theory presented attempts to contain all of the successful results of quantum mechanics and electromagnetic theory in the low energy region of measurements within the microscopic domain, as well as agreeing with the standard predictions of phenomena in the domains corresponding to laboratory dimensions and astronomical dimensions. However, in the high energy region, within the microscopic domain, the theory makes predictions that either are not made at all or are made in a mathematically unsatisfactory manner by the present-day theory which attempts to incorporate the quantum theory of measurement with special relativity theory.

To sum up, the theory to be exploited in this article is based on three essential axioms: (1) the principle of relativity, (2) the generalized Mach principle, and (3) the correspondence principle.

3. *The General Mathematical Structure and Philosophical Implications*

3.1. *The Symmetry Group from Axiom 1 and the Fundamental Field Variables*

The principle of relativity implies a fundamental description of matter that necessarily entails *motion*—the only distinguishing feature between the different frames of reference in which the laws of nature are represented. Motion, in turn, is defined most generally as a continuous entity—the continuous differential changes of one set of space-time coordinates with respect to other sets of space-time coordinates. The transformation group that underlies this theory is then, necessarily, a continuous parameter group (Eisenhart, 1933). Since the minimum number of coordinates that are required to express the laws of nature is 4—usually identified with the space and time coordinates (sometimes with the energy and momentum coordinates, e.g., in scattering problems)—the number of essential parameters that characterizes the symmetry group, in its most general form, must be $4 \times 4 = 16$. These might be represented in terms of the 16 derivatives of the coordinates of one reference frame with respect to another, $\{\partial x^{\alpha}/\partial x^{\beta\prime}\}$. In the general case, these derivatives are a function of where they are evaluated, i.e., the space-time is *nonlinear*.

It follows from Noether's theorem (Lanczos, 1966; Bogoliubov & Shirkov, 1959) that conservation laws result in the local limit (where the nonlinear space-time can be approximated by a linear space-time) as a consequence of the covariance of the formalism with respect to *continuous and analytic* transformations with respect to the space-time coordinates. The conserved quantities that are predicted from these equations are among the predictions of the theory that must be correlated with actual observations. Since these theoretical expressions for the conserved properties of a material system exist only if the transformations of the symmetry group are analytic (as well as continuous) functions of the space-time coordinates and the essential parameters (that distinguish one coordinate frame from another) it follows (by definition) that this is a *Lie group*. The symmetry group that underlies relativity theory is then a 16-parameter Lie group. Henceforth it will be referred to as the 'Einstein group'.

As it was pointed out above, when the local domain is approached (corresponding to infinitesimally small space-time intervals) the nonlinear transformations of the Einstein group approach a linear set of transformations. The general set of nonlinear coordinate transformations are those which leave invariant the (squared) interval

$$ds^2 = g^{\alpha\beta}(x)\, dx_\alpha\, dx_\beta$$

In the local limit of 'special relativity', these transformations approach the linear set that leave invariant the (squared) interval

$$ds_{sr}^2 = dx_0{}^2 - dr^2 \qquad (x_0 = ct)$$

Thus, as the local domain is approached, the metric tensor field, $g^{\alpha\beta}(x)$, of the Riemannian manifold approaches a constant diagonal matrix of numbers of the Lorentzian metric (1 −1 −1 −1)—a Euclidean manifold.

In the linear limit where special relativity is applicable, not all of the 16 parameters are independent. This is because of the special property of the linear space that (a) of the 6 space $\leftrightarrow$ space rotations, $\pm\theta_i$ $(i = 1,2,3)$, only the magnitudes of these angles—the Eulerian angles—are independent (i.e., are essential parameters); and (b) of the 6 space $\leftrightarrow$ time transformations $\pm v_i/c$, only their magnitudes are essential parameters—these are the relative speeds between the inertial frames of reference. Thus, in this case, there are a total of 6 space $\leftrightarrow$ space and space $\leftrightarrow$ time parameters. Adding to these the 4 translations in space-time, we arrive at a 10-parameter Lie group—the Poincaré group of special relativity theory. This reduction in essential parameters shows up in the matrix representation of the Poincaré group in the property that the inverse of a transformation matrix is equal to the transposed matrix. Such a symmetry does not exist in the elements of the transformation group of the (more general) nonlinear space-time, i.e., the Einstein group is the full 16-parameter Lie group (Sachs, 1970b).

It is interesting to note at this stage that there is an alternate definition of 'general relativity theory', adopted by many physicists. To many, the theory of general relativity is based, *axiomatically*, on the specification of a special equation (Einstein's field equations);

$$R_{\gamma\delta} - \tfrac{1}{2} g_{\gamma\delta} R = \kappa T_{\gamma\delta}$$

with the solutions, $g_{\gamma\delta}(x)$, to prescribe the properties of space-time.

With this as the starting point, one can look at the symmetry group that underlies the covariance of these field equations without explicitly referring to the invariant metric ds^2 at each space-time point. One need only refer to the general class of transformations that leave this equation unchanged in form, in a global sense. This symmetry group is, generally, an infinite parameter continuous group. It is not, in most general terms, a Lie group.

On the other hand, the theory discussed in this paper starts with the assertion of the *principle of relativity*—not with any specific set of equations. This author believes that the latter view was in fact Einstein's own view. Consider, for example, Einstein's remark (taken from Schilpp, 1949) in reference to these equations:

> 'Not for a moment, of course, did I doubt that this formulation was merely a makeshift in order to give the general principle of relativity a preliminary closed expression. For it was not anything *more* than a theory of the gravitational field, which was somewhat artificially isolated from a total field of as yet unknown structure.'

Since the assertion of the *principle of relativity* has to do with a comparison of the field equations (i.e., laws of nature) between observers—no matter what these laws are referring to—and since the transformations of the space-time coordinates from one observer's frame to another (relatively moving) one, is in terms of the continuous field of coefficients $(\partial x^{\gamma\prime}/\partial x^{\delta})$ which leave invariant the Riemannian quantity $ds^2 = g^{\gamma\delta} dx_\gamma dx_\delta$, the number of essential parameters that specify this symmetry group is the number of independent coefficients $(\partial x^{\gamma\prime}/\partial x^{\delta})$ at each space-time point—16. With the further specification (for the reasons mentioned above) that these must be *continuous and analytic* transformations, the symmetry group that is discussed here is a 16-parameter Lie group (that we have called the 'Einstein group').

It follows from the feature of relativity theory, as a theory of *motion* which must incorporate conservation laws in the local domain, that the basic variables with which the theory must be expressed are continuous and continuously differentiable functions of the coordinates of the underlying space-time. The space time coordinates themselves are not taken to represent the observables—such as the interpretation of the trajectories of point particles in the atomistic theories. They are rather chosen here as a convenient language—a set of parameters that are *used to map* the continuous field variables, which, in turn, are the basic language elements of this theory. Thus, the implication of relativity theory is a formalism that is based on the *continuous field concept*.

A more explicit feature of the structure of the basic field variables of a relativistic theory follows from the algebraic properties of the representations of the underlying invariance group. It was shortly after Dirac's discovery that a relativistic extension of the Schrödinger-type wave equation leads to the *necessity* to extend from a complex scalar field description to a complex multi-component field description—called a 'spinor' field, when Einstein and Mayer (1932) made a very important discovery about the most primitive representation of a relativistic field theory. These authors addressed themselves to the following question: Was the discovery of the spinor representation of the electron equation a consequence of the quantum mechanical nature of the electron or was it a result of forcing an equation to be relativistically covariant, irrespective of quantum mechanics?

To answer the question, they decided to study the structure of the irreducible representations of the Poincaré group, since this is an aspect of the theory that is independent of the detailed structure of any mathematical formalism—so long as it would be relativistically covariant. Einstein and Mayer then found that if only the *continuous transformations* that leave invariant ds_{sr}^2 are maintained (i.e., leaving out the reflection transformations, such as $\mathbf{r} \rightarrow -\mathbf{r}$) then the (real) four-dimensional representations of this continuous group (the Poincaré group) *decompose* into the direct product of two two-dimensional (complex, hermitian) representations—which obey the algebra of quaternions. Thus, they found that the most primitive types of field variables of a theory that is consistent with the principle of relativity are the 2-component basis functions of these two-dimensional quaternion representations of the Poincaré group, rather than the 4-component vector basis functions of the four-dimensional representations of the same group. It then followed from the algebraic properties of the two-dimensional hermitian representations of the group that their basis functions are in fact entities that obey the defining rules of a spinor variable.†

When the theory is then extended from the representations of the Poincaré group (special relativity) to the representations of the Einstein group (general relativity), the geometric features of these representations are altered. *But the algebraic features are not.* Thus, the two-dimensional hermitian representations in special relativity theory which obey the algebra of quaternions and have as basis functions spinor variables, when extended to general relativity are still quaternion representations with basis functions that are spinor variables.

Einstein and Mayer, then, made the very important discovery that the spinor field is the most primitive type of variable to underlie a theory that is to be consistent with the principle of relativity—irrespective of whether the theory is quantized or not! This is the 'most primitive' type of variable because spinor fields can be combined to yield scalar, vector, tensor fields (of any rank) while no other type of variable can be built up to a spinor. The implication here is that a spinor formalism could yield all of the physical predictions that could be predicted by scalar, vector, tensor, etc. formalisms—but it would make *extra predictions* that have no counterpart in the less general formulations of a covariant theory. A well-known example of the latter point is Dirac's discovery of the energy term that entails a coupling between the spin of an electron (in terms of its magnetic moment) and an external magnetic field. This 'observable' has no counterpart in a scalar, vector or tensor representation of a relativistic field theory.

Thus we have seen that an important mathematical implication (with physical consequences) of the principle of relativity (Axiom 1) is that the

† The algebra of quaternions was discovered by William Rowan Hamilton in the nineteenth century. Most of his original discoveries were published in the *Proceedings of the Royal Irish Academy*. His collected works in this part of his research were published recently in a single volume (Halberstam & Ingram, 1967). A clear discussion of the defining properties of the basis functions of quaternion fields—the spinor—is given by Laporte & Uhlenbeck (1931).

most primitive type of expression of the theory must be in terms of continuous and continuously differentiable spinor field variables. This conclusion is quite independent of other assumptions that one may wish to impose on the theory, such as the assertions (of linearity, nondeterminism, Hilbert space) of the quantum theory or the assumptions of the continuous field concept of the Faraday–Maxwell theory of electromagnetism.

One other important difference between the field theory discussed here, and particle theories, should be noted. This has to do with the role of the space-time coordinates. As it was indicated earlier, the atomistic theories must superpose a separate space-time coordinate system for each of the constituent particles of a system—implying a $4n$-dimensional space-time in a relativistic theory of particles (or a $(3n + 1)$-dimensional space and time in the Newtonian theory). The latter sets of space-time coordinates *are the observables* in the atomistic theories. On the other hand, the field theory discussed here *uses* only a single four-dimensional space-time in which one *maps* many coupled fields within a closed system. This difference will be shown later (in Part III (Sachs, 1971b)) to play an important role in distinguishing between the explicit structures of the fully relativistic theory and an elementary particle theory such as quantum mechanics for a many-particle system.

3.2. *The Generalized Mach Principle and Nonlinearity*

According to this principle, there is no manifestation of matter that is not expressible in terms of the dynamical coupling between matter and matter. The latter two 'coupled' quantities of matter refer here to two aspects of a single closed system that only *appears* to be disconnected into separate parts in some asymptotic limit of sufficiently small energy-momentum transfer among the components of the closed system. But it is important to keep in mind that, within this approach, the separation can never be exact (i.e., the 'free particle' limit does not exist within this theory).

The implication here is that the simplest expression of the theory that fully exploits this idea must be in terms of at least two coupled equations. Let us refer to them, symbolically at first, as follows:

$$0(1,2)\,\psi^{(1)}(x) = 0 \qquad (3.2.1a)$$

$$0(2,1)\,\psi^{(2)}(x) = 0 \qquad (3.2.1b)$$

According to our conclusion of the preceding section, one of these field variables, say $\psi^{(1)}$, may refer to 'observer' and the other, $\psi^{(2)}$, would then refer to the 'observed'. However, we also concluded that with the generalized Mach principle in a relativistic field theory, it should make no difference as to which aspect of this closed system is called 'observer' and which is called 'observed'. Mathematically, this implies that the form of equations (3.2.1a) and (3.2.1b) should be unchanged under the interchange of these two field variables, i.e.,

$$\psi^{(1)} \leftrightarrow \psi^{(2)} \Leftrightarrow \text{eq. (a)} \leftrightarrow \text{eq. (b)}$$

But if this is so, then 0(1,2) must depend on $\psi^{(2)}$ in precisely the same way as 0(2,1) depends on $\psi^{(1)}$. Since 0(2,1) depends on $\psi^{(1)}$, the actual solution $\psi^{(2)}$ of (3.2.1a) depends on $\psi^{(1)}$. But since 0(1,2) depends on $\psi^{(2)}$, which in turn depends on $\psi^{(1)}$ through equation (3.2.1b), the solutions $\psi^{(1)}$ of equation (3.2.1a) must ultimately depend on $\psi^{(1)}$ itself. Thus the coupled field equations (3.2.1a–3.2.1b) are necessarily nonlinear. The nonlinearity in this formalism is, in fact, a consequence of the elementarity of the interaction, rather than the free particle, according to the generalized Mach principle. This is indeed a fundamental difference between this theory and the basis of atomistic theories, such as classical mechanics or quantum mechanics.

Thus far, we have concluded that a theory which fully exploits Axioms 1 and 2 must be in terms of at least two nonlinear, coupled spinor field equations. We have also concluded that the space-time coordinates—the argument x of $\psi^{(1)}$ and $\psi^{(2)}$, are the same. That is to say, the coupled fields, $\psi^{(1)}$ and $\psi^{(2)}$, are mapped in the same space-time. It is only in a limit of sufficiently small energy-momentum transfer between the components of the closed system that these fields can be treated *one at a time*, i.e., when the equations that they solve can be assumed to be uncoupled. In the latter case, one considers $\psi^{(1)}(x_1)$ and $\psi^{(2)}(x_2)$, where x_1 and x_2 are the space-time points for the assumed separate parts of the system, whose respective descriptions are in terms of the fields $\psi^{(1)}$ and $\psi^{(2)}$. In this *mathematical approximation* for the description of the closed system under study, we can superpose the two uncoupled fields in the eight-dimensional space-time that is spanned by x_1 and x_2. This is in the *linear limit* of the nonlinear field equations (3.2.1a–3.2.1b). The explicit structure of these field equations in describing electromagnetically coupled matter and the linear limit of these equations will be derived in detail in Parts II (see p. 453) and III (Sachs, 1971b). First, however, we will discuss further the interpretation of the solutions of these equations.

3.3. *The Conservation of Interaction*

One explicit way of asserting the elementarity of interaction, which is implied by the generalized Mach principle, is to express this concept in terms of a *law of conservation of interaction*. The explicit function of the field variable that appears in the mathematical expression of this law is to represent a connective relation between the component field solutions $\{\psi^{(i)}\}$ of the coupled nonlinear spinor field equations of the closed system.

Taking the interaction field amplitude Ψ $(\psi^{(1)}, \psi^{(2)}, \ldots \psi^{(n)})$ (for an n-particle system) to also transform as a spinor field variable, the differential form of the law of conservation of interaction can then be expressed in the form of the equation of continuity

$$\partial^{\mu}(\bar{\Psi}\gamma_{\mu}\Psi) = 0 \qquad (\bar{\Psi} \equiv \Psi^{\dagger}\gamma_0) \tag{3.3.1}$$

This equation implies that *within any local observer's frame of reference*, the quantity which is represented by the integral

$$\int \Psi^{\dagger}\Psi\, d\mathbf{r}$$

is constant with respect to the time measure *in this frame of reference*.

With the normalization of the field variable Ψ, the function $\Psi^{\dagger}\Psi$ may then be considered to play the role of a *weighting function*. It is interpreted as relating to the weighting of the total interaction within the closed system, in one space-time. Note that the conservation of interaction weighting does not imply that such a weighting is uniform throughout space-time. It does mean that given a closed system, the mutual interaction has a 'flexible' mapping in space-time that *persists* for all times in any observer's measurements. Any alteration of the environmental conditions in a local region that may be made in an experiment, for example, would give rise to a re-distribution of this weighting within the entire system. But any such alteration *within a closed system* cannot cause the weighting function to vanish, even though it can become arbitrarily weak in particular regions of space.

Consider now a few cases which exemplify the role of the interaction field in physical situations. An interesting case is that of electron–positron annihilation and creation. For if matter should annihilate and be created at arbitrary times, as it is conventionally assumed to happen, then the weighting function $\Psi^{\dagger}\Psi$ would no longer be conserved in time, i.e., in these cases the integral $\int \Psi^{\dagger}\Psi d\mathbf{r}$ would indeed vary with time. This, then, indicates that the field theory discussed in this paper must necessarily *predict* all of the experimental facts that are usually interpreted as pair annihilation and creation—but without actually creating or annihilating matter at any time. These results will indeed be demonstrated in Part IV (Sachs, 1971c).

It is also interesting to examine the interpretation of the conventional description of the hydrogen atom (or many-electron atoms) within the framework of this theory. That is, while the nonlinear field equations for the e–p system approach the exact form of the Schrödinger equation for hydrogen in the nonrelativistic limit, the properties of hydrogen must be interpreted here in terms of the weighting of the interaction between the electron component and the proton component of *the closed system, electron-proton*, rather than considering the two particles as isolated *singularities*, perturbing each other at a distance.

According to this theory, the presence of an electron and a proton in the universe must be accounted for in terms of a *continuous field* that *weights* their mutual interaction. It follows from the solutions of the field equations that the electron–proton interaction is weighted most heavily in the region of space that has the radius referred to as the 'first Bohr orbit'. With this interpretation, then, no reference need be made to the electron and proton as isolated entities. In this way, the 'atom' can be represented with a formalism that is based entirely on the field concept, and is in strict accord with

Axioms 1 and 2—*the principle of relativity* and *the generalized Mach principle*.

It should be emphasized at this point that the discreteness of physical observables, such as atomic energy levels, is, within this theory, only an *apparent discreteness*. For it is only within an approximation to the exact nonlinear equations of the theory that one arrives at the linearized eigenvalue equations for the atomic system—thereby leading to the predicted (apparent) discreteness of atomic energy levels (Axiom 3). Thus the proposed theory predicts that these energy levels are not, in principle, discrete but do indeed have a finite width which arises from the physical coupling with the rest of the closed system. Since, according to Axiom 2, this coupling can never be totally 'off', the line widths for the spectral distribution of atomic energy levels, for example, can never be zero. That is to say, the values for the properties of matter (of any quantity) have a continuous (rather than a discrete) set of values, within the present theory.

To exemplify further the contrast between the aspects of continuity and discreteness in physically measured properties, consider the operation of a Geiger counter. At first sight, the operation of this device appears to entail the occurrence of *discrete energy bundles* entering the counter *at random times*. One then associates the 'clicks' of the counter with the existence of discrete things that move about in an acausal fashion.

It is clear that this data does not compel one to assert that discreteness is a fundamental ingredient of the underlying theoretical basis for these phenomena. For the counter is not more than an electronic instrument with a voltage bias that is set by the experimenter at a convenient level in order to discharge electric energy whenever its interaction with some other electrical source exceeds some predetermined threshold voltage. As the voltage bias (and therefore the threshold for a 'click' to occur) is lowered, more clicks will be heard in a fixed amount of time. In the limit of no bias voltage, the discrete set of clicks will wash out into a steady background 'noise'. That is to say, in this limit, the 'signal-to-noise' ratio should be reduced to unity. Now, to interpret this 'noise' as a random superposition of the effects of uncoupled things is to assume an ideal limit that cannot be directly verified in experiment; it can only be postulated! Indeed, this is the postulate of atomism. Still, the actual data does not compel one to adopt this model as the unique explanation.

As we have emphasized earlier, the property of discreteness is *abstracted* from the measurements of continuous (though peaked) values for the conserved properties of the system. Whether the underlying abstract idealization is based on a theory that matter is fundamentally discrete or a theory that it is continuous can only be tested indirectly. That is to say, these are theoretical abstractions that can only be postulated and logically and mathematically exploited; they are not directly observable assertions.

To sum up, the clicks of the Geiger counter, the optical spectrum of a radiating gas, the collision experiments of Franck and Hertz, etc. clearly indicate the peaked nature of the *interaction weighting* for the corresponding coupled systems. But the results of these experiments do not necessarily require the *conceptual base* of the quantum theory for an explanation. Indeed, the nonlinear field theory which fully exploits the *principle of relativity* by starting with the elementarity of the interaction (that is, the elementarity of the *closed system*), rather than that of the particle of matter (the *thing-in-itself*) does describe the same data in terms of a continuous, though peaked, set of values.

These results are derived here from general expressions for the conserved properties of a system that follows from the invariance properties of the continuous field description (Noether's theorem). This field theory is expressed in terms of a set of coupled, nonlinear field equations that do not *generally* have the eigenfunction structure of the quantum formalism. The peaked nature of the predicted values for the conserved properties of the system follow here in the low energy limit, where the formalism approaches that of the quantum theory, as a *linear approximation*.

Finally, an important question that relates to the interaction field as a weighting function has to do with the interpretation of the *Pauli Exclusion Principle*, and with its derivation, from the proposed theory. That is, how, within the framework of a single field approach, does one interpret the statement of this principle, which appears to entail the correlation of the positions and momenta of a many-particle system? To answer the question, it will be shown in Part III (Sachs, 1971b), and in Sachs (1963), that as a consequence of the self-consistency *and full nonlinearity of the field equations* for interacting particles, an interaction field amplitude Ψ satisfying the continuity equation (3.3.1), vanishes identically when two of the coupled fields, out of an n-field system, have (1) the same inertial mass, (2) a mutually repulsive interaction, and (3) are in precisely the same state of motion. In other words, the interaction between two identical particles, each in the same state of motion, makes no contribution to the total weighting amplitude for the closed system (although their separate interactions with the rest of the system do contribute to the observables). The physical implications of this result are identical with those of the *Pauli Exclusion Principle*.

In conclusion, it should be emphasized that *the law of conservation of interaction* is not an extra postulate of this theory. It is, rather, logically necessitated by Axioms 1 and 2 which underlie the description.

3.4. *Determinism*

An important philosophical implication of the relativistic field theory which incorporates the Generalized Mach Principle (Axioms 1 and 2) is that such a theory is necessarily based on the concept of determinism. This underlying aspect of the theory is, of course, in sharp contrast with a fundamental feature of the quantum theory which asserts that nature is intrinsically nondeterministic. Since this is an important philosophical feature of the proposed field theory which contrasts with quantum field theory, we will now be more explicit about the precise meaning that is given to the term 'determinism', as it is used in these theories.

What is *not* meant by this term is simply an ordering of events *in time*—although the latter is related to a special case of the general concept. In the latter special case, one says that there is a law of nature, e.g., wave mechanics, that is based on an equation of motion

$$H\psi = i\hbar\frac{d\psi}{dt}$$

The term 'motion' is defined here only in terms of the variation of certain variables with respect to the time coordinate. The integration of this equation and the knowledge of the value of ψ at some initial time then tells us how ψ is ordered along the time axis. Another well-known example is Newton's second law of motion

$$m\frac{d^2\mathbf{r}}{dt^2} = \mathbf{F}$$

Knowing two boundary conditions here (initial position and velocity) the integration of this equation gives $\mathbf{r}(t)$—the trajectory of the particle with mass m.

In our more general meaning of the term, these are only special cases of determinism. When we say that a theory is deterministic, we are referring to the feature that all of the variables that describe the considered system are 'predetermined'. That is to say, it is the assertion of a deterministic theory that there exists a complete description of the system, which is precisely mapped out in the most convenient parameter space. The usual parameter space that is chosen is the 4-parameter space-time coordinate system. Sometimes an energy-momentum coordinate system is more convenient (e.g., in scattering problems).

In quantum mechanics it is said that the states of motion of a material system are not predetermined. It is true that the Schrödinger equation is a perfectly ordered description and does indeed have a predetermined set of solutions. But this is only meant here in the sense that these are not more than the elements of *a language* that a macroscopic measuring apparatus would use in reporting about the physical properties of some microscopic system that is under observation—*a language that has only to do with probability statements*. The salient point of this discussion is the assertion of the quantum theory that the *fundamental variables* which relate to the states of motion of elementary particles have only to do with probability statements that are made by a particular measuring apparatus about a particular microscopic system. These conclusions then lead to the feature of the quantum theory: that the basic properties of matter are not predetermined—that they depend instead on the nature of the measurement—and that all the values of these properties are not determinable simultaneously, arbitrarily or precisely. It is concluded in the quantum theory that the accuracy to which some of these variables can be known depends in a reciprocal way on the precision with which other of the properties can be known at the same time, *from a measurement carried out with a macroscopic apparatus* (the *Heisenberg Uncertainty Principle*). Thus we see that the quantum theory is fundamentally subjective in nature and is nondeterministic. All other theories in physics (preceding and contemporary) are objective in nature (i.e., the properties of matter are independent of the conditions under which they are measured) and deterministic.

The important question is: Is the nondeterministic interpretation of the equations of quantum mechanics (a formalism that is certainly *empirically* correct in the low energy region) *necessary* in order to understand microscopic systems?

It will be shown in Part III (Sachs, 1971b) that one can, in fact, arrive at the mathematical formalism of nonrelativistic quantum mechanics as an asymptotic approximation for a continuous field formalism with an entirely different conceptual basis—one that depends, among other things, on the concept of determinism, in the broad sense of the term that has been discussed above. Thus, the answer to this question is that, indeed, nondeterminism is *not* an *a priori necessary* conceptual ingredient in the laws of nature (Sachs, 1970d).

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